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Thermal Properties
and
Heat Transfer Characteristics
of Marsh Grapefruit

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Thermal Properties and Heat Transfe Characteristics of Marsh Grapefruit

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SUMMARY

Investigations of the thermal properties and heat transfer chara
teristics of Marsh grapefruit were conducted on 10 test fruits fro
each of five maturity groups. The harvest dates for maturity grou
were: December 1963, April 1964, October 1964, February 1965, an
May 1965. The investigations included tests to determine (1) effe
tive thermal diffusivity of the whole fruit, (2) thermal conductivi
of the rind and juice-vesicle components, (3) moisture conten
(4) specific gravity, and (5) internal-temperature response—fund
mental to response at specified conditions.

Of the properties measured, significant relations were found betwe
rind thickness, moisture content of both rind and juice vesicle
thermal conductivity of both rind and juice vesicles, and specif
gravity. By visual inspection these properties also appear to correla
with harvest season. As the harvest season is extended, the fru
ripens. During the ripening process, the fruit becomes more dens
its rind becomes thinner, and the rind moisture content decrease
These changes are consistent with a measured increase in effecti
thermal diffusivity as the fruit ripens. However, cooling rate is n
significantly affected by differences in the maturing and ripeni
of the fruit.

INTRODUCTION

Refrigeration is the basic means for postharvest protection again
decay and deterioration of horticultural crops. The extraction
heat from these living biological products slows their respirato
activity, reduces water loss, decreases the chance of invasion of ne
decay, and inhibits or slows the growth of incipient infections. T
optimum rate and magnitude of heat extraction vary dependi
upon the physiological nature of the product. Some products a
more perishable than others; thus they need to be cooled more rapidl
Marsh grapefruit, for example, differs from other citrus fruits in i
physiological response to temperature. It also responds different
at various stages of maturity. In tests with Marsh Seedless a
Ruby Red grapefruit, Chace and coworkers (*3*) found that the mo
desirable transit temperature for early-harvested fruit was 60°
More mature midseason and late-season fruit kept best in trans
at 50°. Their findings, substantiated by an abundance of previo

research cited in their report, point out the need for postharvest, preshipment conditioning commensurate with the particular fruit requirements.

In a 1958 report on hydrocooling Florida citrus, Grierson and Hayward (*4*) stated that—

> "The increasing use of automated methods, combined with the danger from such endemic post-harvest diseases as stem-end rot and *Penicillium* mold, make the use of such packing methods hazardous unless efficient refrigeration, post-harvest fungicides, or a combination of these two protective methods is used."

They further reported finding increased susceptibility to decay and rind injury on hydrocooled fruit. Yet, precooling, whether through hydrocooling or through the use of an air-cooling system, is one of the essential means of conditioning fruit in preparation for shipment. Precooling implies rapid heat removal, which may be done at the packinghouse in bulk, in pallet boxes, or in shipping containers. Generally it is done before shipment. Sometimes it is done after the fruit has been loaded into trucks or railcars. Either way, optimum precooling, i.e., the removal of a specified predetermined quantity of heat in a given period of time, depends upon knowledge of the thermal properties and characteristics of the fruit in question. This basic information will benefit the citrus industry in that it will eventually lead to the development of more efficient and effective precooling systems.

Published values of thermal properties of grapefruit, reviewed by Bennett (*1*), are inconclusive and questionable. Smith and coworkers (*10*) and Perry and coworkers (*6*) reported values of thermal diffusivity of Marsh grapefruit that are accepted by the authors to be accurate, and they are therefore compared with the results of the work reported herein.

This research was conducted to evaluate the basic heat transfer properties and characteristics of Marsh grapefruit and to investigate the possible relation of these properties to moisture content, density, and harvest season. Specifically, the research was designed to yield the following data:

1. Experimental temperature distribution.
2. Effective thermal diffusivity.
3. Thermal conductivity of the rind and juice vesicles.
4. Moisture content of the rind and juice vesicles.
5. Specific gravity of the whole fruit.
6. Correlations of the foregoing with seasonal effects.

INVESTIGATIONS

Test Fruit

Samples of Marsh grapefruit were harvested in December 1963, in April and October 1964, and in February and May 1965, from commercial groves in Indian River County, Fla. Five maturity groups, numbers 1 through 5, referred to hereinafter chronologically, correspond with the foregoing harvest dates. Ten test runs were

made for each maturity group on the basis of expected experimenta variation of runs within a group.

The fruit was washed and waxed with a solvent-type wax t prevent loss of moisture during the short storage period before testing Storage was at 50° F.

Experimental Procedure

Each test fruit was weighed and its diameter measured at severa radial points in both the equatorial and polar planes. The fruit wa brought to a uniform temperature of 85° F., and then it was immerse in an agitated water bath held at a constant temperature of approximately 35°. Fruit temperature was measured at ¼-inch interval along the radius in the equatorial plane by means of a thermal prob constructed of 36 a.w.g. (American wire gage) copper-constanta thermocouple wire connected to a 24-point recording potentiomete The probe consisted of 12 individual thermocouples and was o sufficient length to permit insertion along the entire length of th diameter. This procedure compensated for conduction error, becaus the heat of conduction along the wire tended to flow in an opposit direction from that of the heat flux in the fruit. Surface temperatur and temperature just beneath the rind were measured with individual thermocouples (fig. 1). The test fruit was cooled to a cente temperature of approximately 40°, removed from the water bath

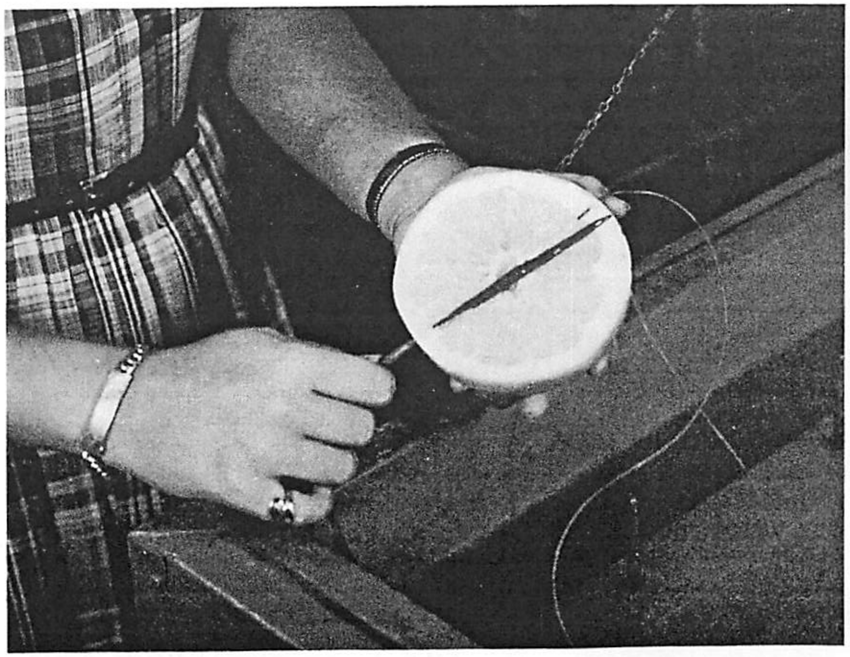

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FIGURE 1.—Thermal probe positioned in test fruit. Note thermocouple to measur temperature at interface between rind and juice vesicle.

and reweighed for further tests. The test apparatus is shown in figure 2.

Specific gravity was measured by the water-displacement technique. Thermal conductivity and moisture content of the rind and the juice vesicle were measured from samples of each specimen.

The moisture content was measured by weighing the sample, extracting the water in a vacuum oven at 140° F. for 48 hours, then weighing the residual solids. Percentage moisture content was calculated on the basis of the wet weight of the sample.

The thermal conductivity was measured by use of an adaptation of the Fitch method (fig. 3) for measuring thermal conductivity of poor conductors. Equipment used included the adapted test unit, sensitive temperature controller, stopwatch, agitator, low-resistance microammeter, laboratory potentiometer, micrometer (fig. 4), and pressure meter (fig. 5). The experimental technique of Bennett and coworkers (*2*) was used.

Test specimens were removed from the fruit by use of a sharp-edged, hollow, stainless steel tube (fig. 6) having an inside diameter of 1.25 inches. The sample was cut to the same diameter as the heat sink to eliminate the possibility of heat energy radiating to the sink from the plate. Sample pressure was held standard at 1 p.s.i. (pound per square inch) for the rind and 0.5 p.s.i. for the juice vesicles. Juice-vesicle samples were wrapped in thin polyethylene to reduce moisture evaporation and juice losses.

Rind thickness was obtained from an average of nine micrometer readings taken before and after each run (fig. 7). An average of five measurements was used for juice-vesicle thickness.

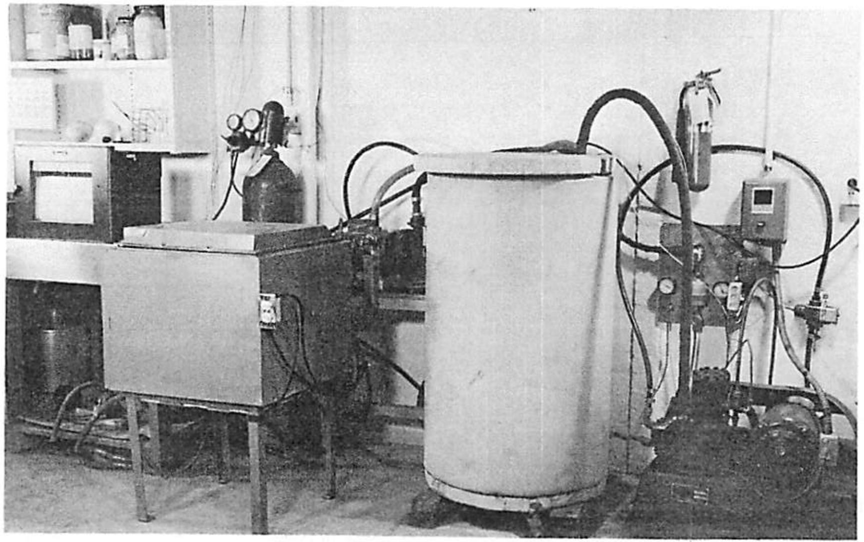

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FIGURE 2.—Equipment for evaluating temperature distribution and effective thermal diffusivity of Marsh grapefruit.

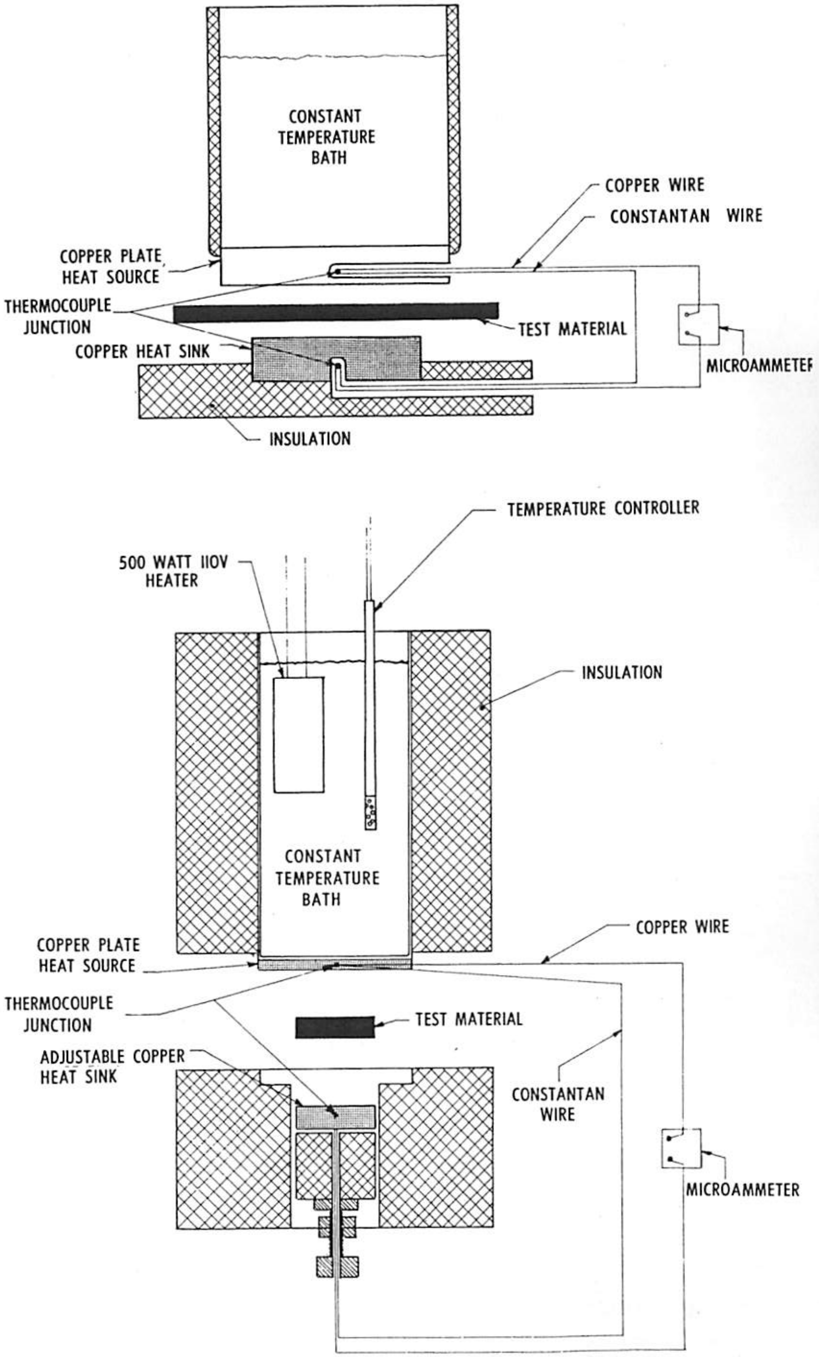

FIGURE 3.—Schematic cross-section of the standard Cenco-Fitch test unit f measuring thermal conductivity of poor conductors (above) and the adapt test unit (below).

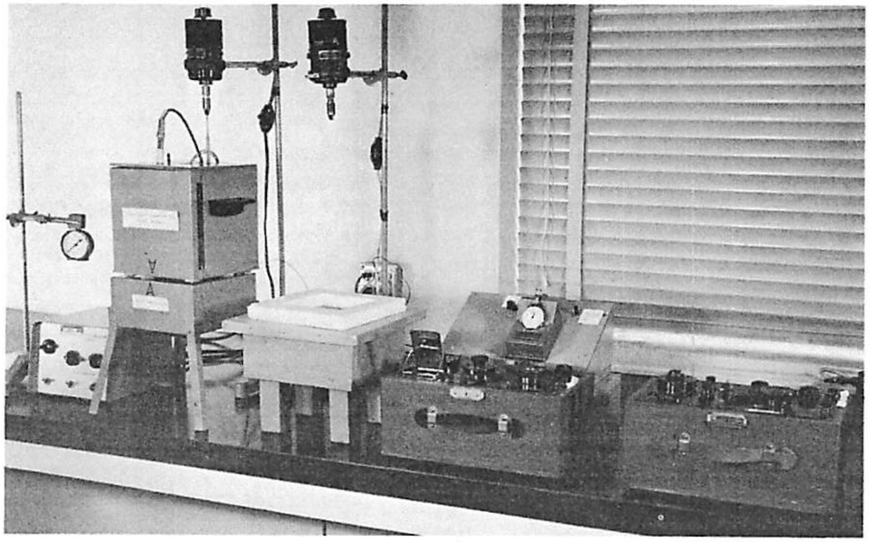

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FIGURE 4.—Laboratory equipment and instruments used to measure thermal conductivity of Marsh grapefruit rind and juice vesicles.

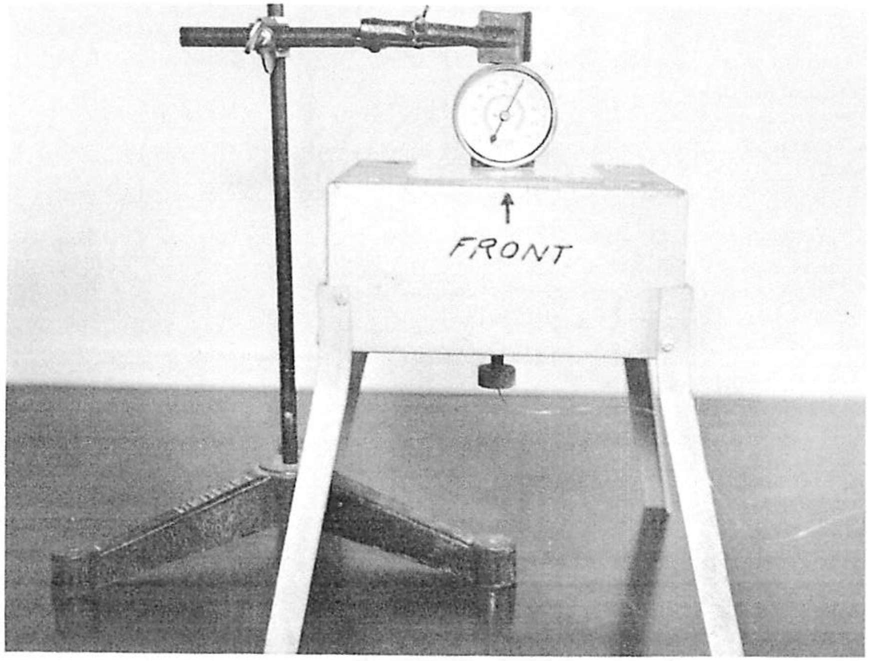

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FIGURE 5.—Pressure tester positioned over test sample in base of unit. Each sample was subjected to a standard predetermined pressure to minimize contact resistance.

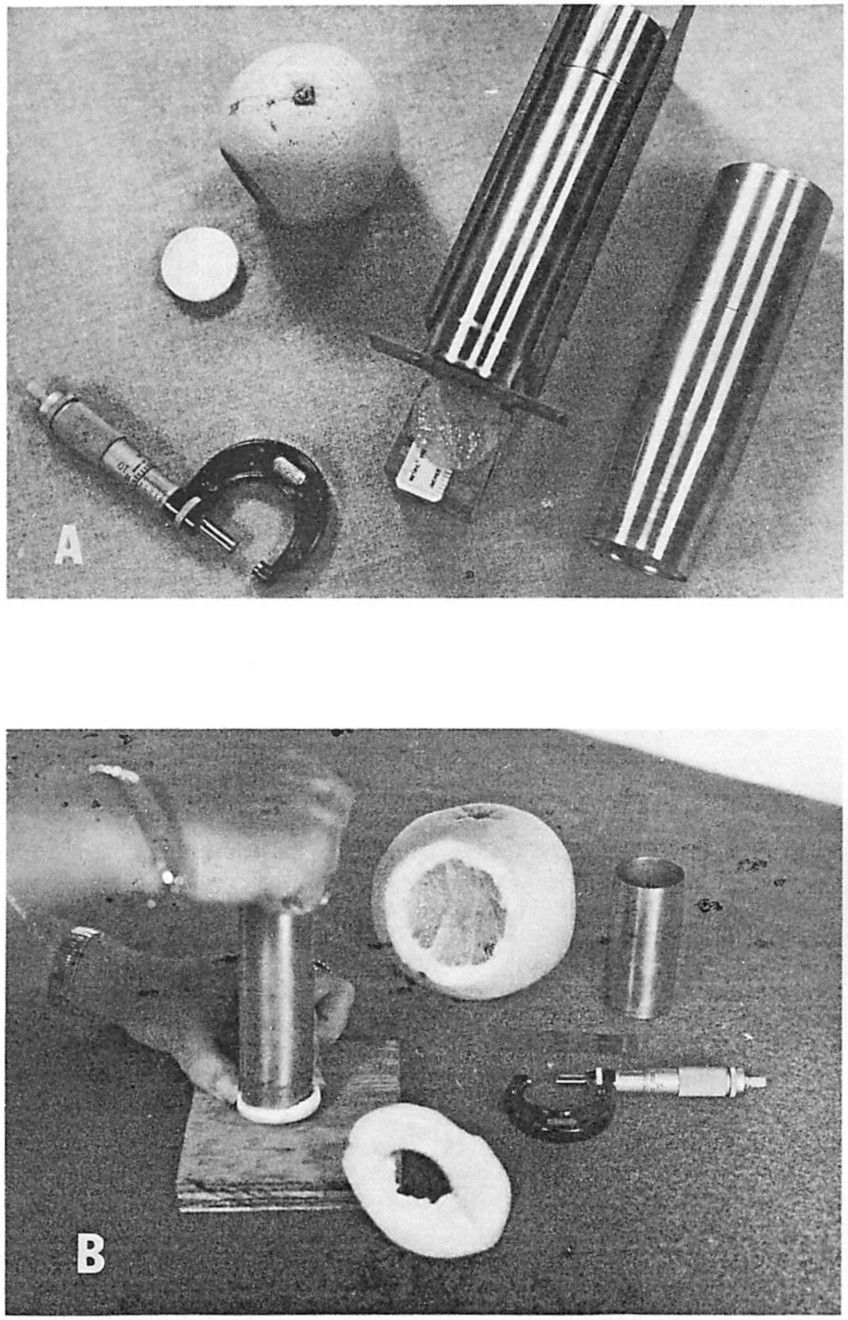

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FIGURE 6.—Equipment (*A*) and procedure (*B*) for obtaining rind and juic vesicle test samples from the fruit for measuring moisture content and therm conductivity.

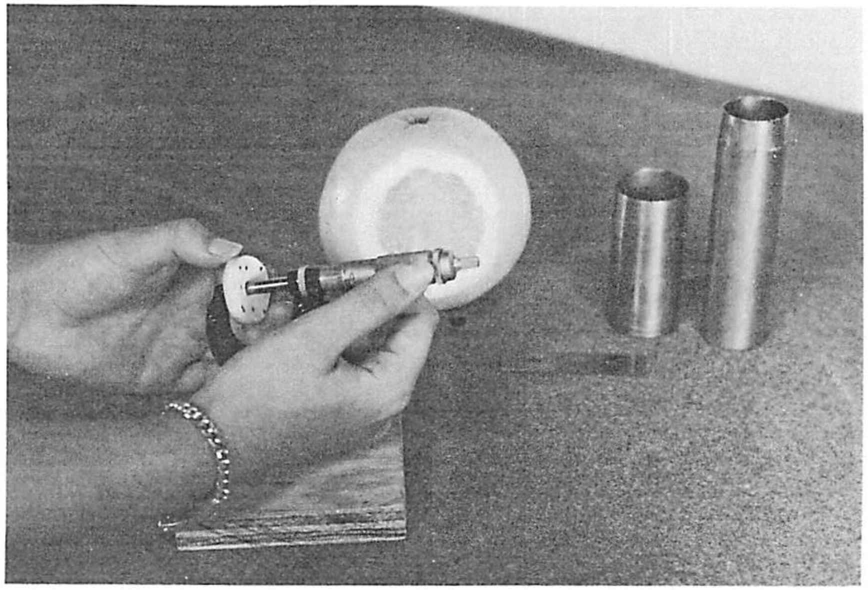

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FIGURE 7.—Measuring rind thickness of Marsh grapefruit.

ANALYSIS AND RESULTS

Effective Thermal Diffusivity

Theoretical equations introduced by Fourier some 145 years ago opened the way for the mathematical solution of problems involving conduction heat transfer in homogeneous solids. He developed equations in the form of power series for objects having such basic shapes as the sphere, the cylinder, and the rectangle. Substitution of the characteristic dimensionless ratios for temperature, time, and object dimensions into the partial differential equation that describes temperature history within a particular geometric shape enables solution of the temperature response as related to time and location within the object for a prescribed heating or cooling situation. And substitution of experimentally measured values of the temperature response for the appropriate time and location within a substance enables evaluation of the substance's thermal diffusivity, one of the basic heat transfer properties.

From Fourier's basic equations, several investigators have developed methods for the analysis of transient state heat transfer problems for particular object geometries. Most of these investigators are cited by Pflug and coworkers (*7*) in a valuable work describing methods for developing temperature-time curves for the three common geometries, the sphere, the infinite plate, and the infinite cylinder. However, the similarity of the various equations permits the temperature response for all geometries to be described in terms of a single

general form. If only the first term of the series and the straight-lin approximation are used, a simple expression may be written. Wit application of the notation of Smith and coworkers (*10*), the equatio takes the form

$$T=Ce^{-M_1{}^2Fo} \qquad ($$

where T is the dimensionless temperature ratio, $\frac{t-t_s}{t_i-t_s}$ (subscript denotes initial temperature and subscript s denotes surface temper ture), C and M_1 are functions of the object geometry, propertie and boundary conditions. The Fourier number, *Fo*, is a dimensio less ratio expressed in terms of $\alpha\tau/\ell^2$, where

α=thermal diffusivity, sq. ft. per (hr.)
τ=time, hours
ℓ=characteristic length, feet

When equation (1) is used to solve for the theoretical temperatu response of an object in a specified heat transfer situation or experimentally determine heat transfer properties of an object, it necessary to evaluate M_1. The transcendental equation for a sphe is

$$N_{Bi}=1-M_1 \cot M_1 \qquad (2$$

where the Biot number, N_{Bi}, is a dimensionless ratio that describ the surface heat transfer capability as related to the heat transf property and dimension of the object. It is mathematically expresse by $(h \times r)/k$, where

h=surface heat transfer coefficient, B.t.u. per (hr.) (sq. ft.) (° F.)
r=radius of sphere, feet.
k=thermal conductivity of object, B.t.u. per (hr.)(sq. ft.) (° F. pe ft.)

Equation (1) is useful only when there is a temperature gradie within the object under consideration. There is a limiting boundar condition (surface heat transfer capability) that results in a negl gible temperature gradient within an object being heated or coole For example, a small copper sphere being cooled in relatively sti air will have an imperceptible thermal gradient along its radius any time, which causes the Biot number to become infinitely smal Thus, M_1 approaches zero and T becomes equal to C for all value of time. When the temperature gradient is negligible, an evaluatio of thermal diffusivity through the use of equation (1) is not possibl

As a contrast, a substance having relatively poor thermal conduct ance will exhibit a marked thermal gradient when heated or coole if its surface suddenly becomes equal to the temperature of th surrounding fluid. Smith and coworkers (*10*) and Pflug and coworke (*7*) describe the case for conditions of finite surface heat transfe resistance.

When Marsh grapefruit is suddenly immersed in a well-agitate water bath, the temperature on its surface can be assumed to rapidl approach the temperature of the cooling water. Thus, effectiv thermal diffusivity can be evaluated on the basis of fruit temperature response. Because of the finite heat transfer resistance at the surface, the reciprocal of the Biot number is so small it can be neglected. However, acurate evaluation by the conventional method requires not only that specified boundary conditions be rigidly adhered to but also that the substance be homogeneous and that it conform to one of the conventional object geometries.

Whereas, for Marsh grapefruit, the specified boundary conditions can be easily satisfied, the requirement for homogeneity and object geometry is more difficult to satisfy. Actually, Marsh grapefruit is composed of constituency having widely varying properties and physical structure. In these evaluations, it must therefore be considered as a pseudohomogeneous material. In addition, it is shaped as an oblate spheroid. Failure to account for its departure from sphericity causes an error that is proportional to the magnitude of departure. Normally the equatorial diameter of Marsh grapefruit is 10 to 15 percent larger than the polar diameter. Therefore, the more accurate evaluation of effective thermal diffusivity of Marsh grapefruit is made by conceding its pseudohomogeneous composition and by making the necessary geometry correction.

Smith and coworkers (*10*) developed a technique of evaluating thermal diffusivity that corrects for deviation from the conventional shapes. The technique incorporates a geometry index, G, into the basic Fourier equation for a sphere. The geometry index was obtained "from a measure of two orthogonal areas of the shape." Use of this technique provides a means for a more accurate evaluation of effective thermal diffusivity of Marsh grapefruit than has previously been available. It is a significant breakthrough for investigators of thermal properties of biological materials having anomalous geometries. The equation, as presented by Smith and coworkers, is

$$\alpha=\frac{l^2}{M_1^2}\left[\frac{lnT_1-lnT_2}{\tau_1-\tau_2}\right]. \qquad (3)$$

The ratio of the differences, lnT vs. τ, is the slope, β, of the cooling curve for the corresponding time interval, or

$$\beta=\frac{lnT_1-lnT_2}{\tau_1-\tau_2}. \qquad (4)$$

Therefore, the equation may be written

$$\alpha=\frac{l^2\beta}{M_1^2}. \qquad (5)$$

When Marsh grapefruit is cooled in an agitated water bath, the surface heat transfer resistance is negligible. Hence,

$$M_1^2=G\pi^2. \qquad (6)$$

From 13 test runs, Smith and coworkers (*10*) measured an average thermal diffusivity of 3.63×10^{-3} sq. ft. per hr. for Marsh grapefruit.

This value compares favorably with 3.54×10^{-3} sq. ft. per hr. fro 15 tests runs as measured by Perry and coworkers (*6*) on Mars grapefruit. By substitution of the values of slope, characteristi length, and function of geometry index, M_1, (as measured for eac test fruit) into equation (*5*), the effective thermal diffusivity w evaluated for each individual test run. These values, averaged b maturity group, are listed in table 1. The method of evaluating slo is described in a later section (p. 22).

TABLE 1.—*Experimentally obtained effective thermal diffusivity Marsh grapefruit; evaluated from temperature response at the cente one-half the radius, and three-fourths the radius*

[Average of 10 runs for each maturity group]

Maturity group [1]	Center		One-half radius		Three-fourths radius	
	Effective thermal diffusivity	Coefficient of variation	Effective thermal diffusivity	Coefficient of variation	Effective thermal diffusivity	Coefficient of variation
	10⁻³ sq. ft./hr.	*Percent*	*10⁻³ sq. ft./hr.*	*Percent*	*10⁻³ sq. ft./hr.*	*Percent*
1	3.44	5.43	3.76	5.97	4.11	6.
2	3.15	5.12	3.37	5.42	3.59	5.
3	2.74	5.00	2.93	5.39	3.05	5.
4	2.95	5.10	3.15	5.45	3.36	5.
5	3.09	5.47	3.37	5.96	3.64	6.

[1] Harvest dates for maturity groups are given on p. 2.

If sufficient time is allowed for the rate of temperature change to b uniform throughout a solid homogeneous sphere, the slope of th linear temperature response will be equal at all points along an radial coordinate. Because of grapefruit characteristics and of ambiguity associated with heat conduction along the probe, temperatur response of Marsh grapefruit produces a small difference in slope a the three points along the radius. This difference is reflected in th resulting values of effective thermal diffusivity as evaluated on th basis of temperature response at the center, one-half the radius in th equatorial plane, and three-fourths the radius in the same plane. W believe the more accurate values are taken at three-fourths the radiu which is approximately the point of mass-average temperature.

The effective thermal diffusivity of Marsh grapefruit might b arbitrarily assumed to vary with temperature in somewhat the sam proportion as water; i.e., a decrease of about 10^{-5} sq. ft./hr. for eac ° F. temperature reduction in the range considered. The averagin effect of the above described method—that of evaluating effectiv thermal diffusivity based on time-temperature response within homogeneous sphere in an agitated bath—negates this phenomeno But this method yields values for specified times corresponding t known values of fruit temperature, which may be statistically analyzed for correlations of temperature on thermal diffusivity.

From 631 observations in 50 test runs, the linear correlation of mass average temperature with effective thermal diffusivity, based on tem perature response at the center, was found to be

$$t = -7.42 + 13592.24\alpha. \qquad (7)$$

From a regression analysis, in which five types of curves were tested, this was the best fitting linear response obtained. Average values so obtained are approximately 22 percent greater than those found by applying the geometry correction to the first-term approximation. Interestingly, the deviation is of the same order of magnitude and in the same direction as that found when the first-term approximation is used without applying the geometry correction described above. The solid line of figure 8 illustrates the correlation as it stands without applying the geometry correction. A constant reduction of 22 percent over the temperature range yields a set of values comparable with those obtained by the geometry correction technique and that show the influence of temperature on diffusivity. The corrected values are illustrated with the dashed line in figure 8.

Dimensions and physical properties, averaged by maturity groups, are listed in table 2.

Evaluation of effective thermal diffusivity for the whole fruit was made on the basis that the fruit rind and juice-vesicle components comprise one homogeneous mass constituency. Actually, the two components contrast sharply in their texture and composition. The rind is composed of what is called the albedo and the flavedo. Th albedo is a spongy layer of loosely arranged cells with many gas-fille intercellular spaces. The flavedo, or epidermal layer, contains numerous oil sacs and is more dense than the albedo. The juice-vesic component is composed of numerous tightly arranged liquid-fille sacs. The density of the rind is about one-half that of the juice vesicle

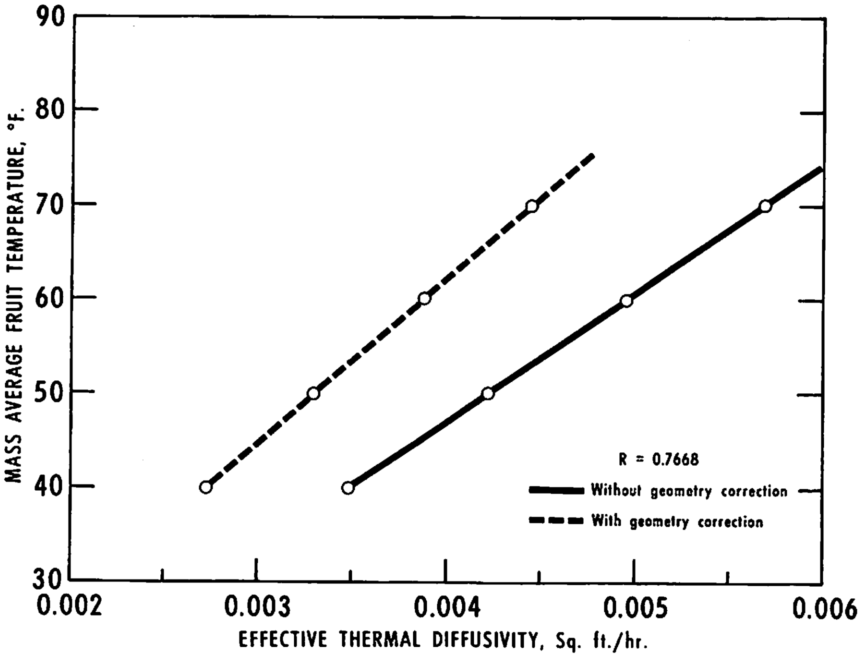

FIGURE 8.—Influence of temperature on experimentally evaluated effective thermal diffusivity of Marsh grapefruit.

TABLE 2.—*Diameter, specific gravity, thickness of rind, and moistu content of rind and juice-vesicle components for Marsh grapefruit*

[Average of 10 runs for each maturity group]

Maturity group [1]	Equatorial diameter	Coefficient of variation	Polar diameter	Coefficient of variation	Specific gravity (whole fruit)	Coefficient of variation
	Inches	*Percent*	*Inches*	*Percent*		*Percent*
1	4. 134	3. 3	3. 806	7. 9	0. 84	3.
2	4. 162	2. 8	3. 644	5. 4	. 86	3.
3	3. 895	3. 6	3. 431	6. 0	. 82	2.
4	3. 975	2. 5	3. 525	3. 5	. 85	2.
5	4. 038	5. 5	3. 557	5. 6	. 88	1.

Maturity group [1]	Rind thickness	Coefficient of variation	Moisture content of rind	Coefficient of variation	Moisture content of juice vesicle	Coefficient of variation
	Inches	*Percent*	*Percent*	*Percent*	*Percent*	*Percent*
1	0. 306	13. 5	79. 3	3. 7	87. 8	0.
2	. 252	13. 4	77. 2	2. 0	87. 3	1.
3	. 307	21. 6	79. 4	2. 6	88. 4	.
4	. 253	15. 3	79. 6	1. 3	88. 8	.
5	. 224	15. 5	77. 8	1. 9	88. 9	1.

[1] Harvest dates for maturity groups are given on p. 2.

However, the difference in density does not affect the exper mentally evaluated thermal diffusivity. As Perry and coworke (*6*) described it, "in a given material where density changes becaus of changes in porosity, the thermal conductivity is approximatel proportional to density, so that the diffusivity remains about co stant." They confirmed their theory by a numerical solution involvin two concentric spheres of known thermal properties. Further suppo of their theory was gained from the results of a more thorough scrutin of the two separate components. The thermal conductivity, densit and specific heat were expressed in terms of a calculated diffusivit for the rind and for the juice vesicles separately to provide a co parison of the two respective values for each maturity group. Th results are listed in table 3.

The equation for determining the calculated thermal diffusivit α, is

$$\alpha = \frac{k}{\rho c_p}$$

where

k=thermal conductivity, B.t.u. per (hr.) (sq.ft.) (° F. per ft.)
ρ=density of substance, lbs. per (cu. ft.)
c_p=specific heat of substance, B.t.u. per (lb.) (° F.)

Specific heat data were calculated from Siebel's equation[1] on the basis of moisture-content data reported in table 2. Density values were obtained by Otto Jahn,[2] during the 1964–65 season, from fruit other than that used in this work. Thermal-conductivity data were experimentally evaluated as described later (p. 15).

The similarity in thermal diffusivity of the rind and juice-vesicle components calculated by equation (*8*) substantiates the logic of Perry and coworkers (*6*). Values so obtained are noticeably larger than those reported in table 1. This difference may be attributable to the influence of temperature on thermal diffusivity; it may be the result of convection heat transfer in the rind and juice-vesicle samples during thermal-conductivity tests; or possibly it may be the result of both. Table 1 values are based on measurements taken at an average temperature of approximately 55° F. Table 3 values are based on measurements taken at logarithmic mean temperature of 80° for the rind and 88° for the juice vesicle. Temperature effect is seen in figure 6. The texture and moisture content of the specimen is such that thermally induced fluid movement within the intercellular spaces of the rind and within each individual juice vesicle sac can be significant. The high temperature gradient across the sample is certain to induce some fluid movement. Consequently the convection component could cause the values to be larger than they would be if heat transfer were by conduction only.

TABLE 3.—*Specific heat, density, and calculated thermal diffusivity of the rind and juice-vesicle components of Marsh grapefruit*

Maturity group[1]	Specific heat,[2] B.t.u./lb./°F.		Density,[3] lb./cu. ft.		Thermal diffusivity,[4] 10^{-3} sq. ft./hr.	
	Rind	Juice vesicle	Rind	Juice vesicle	Rind	Juice vesicle
1	0. 835	0. 901	35. 1	63. 4	4. 80	4. 50
2	. 818	. 898	36. 1	63. 3	4. 65	4. 94
3	. 835	. 906	35. 2	63. 7	4. 75	4. 95
4	. 837	. 910	35. 6	62. 9	4. 79	4. 94
5	. 822	. 911	38. 7	62. 7	4. 34	4. 45

[1] Harvest dates for the maturity groups are given on p. 2.
[2] Based upon average of 10 runs within each maturity group.
[3] Personal communication from Otto Jahn.
[4] Calculated from experimentally obtained thermal conductivity.

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[1] c_p=0.008 X percent moisture content +0.20.
[2] Personal communication.

Characteristic Thermal Conductivity of Rind and Juice Vesicle

Characteristic thermal conductivity for the rind and juice-vesicl components was evaluated by use of the equation

$$k=\frac{C}{Lb} \qquad (9$$

where the constant, C, is a measure of the heat-retaining capacit of the copper sink that is located in the base of the Cenco-Fitcl apparatus. It has a value of 3.3 B.t.u. per (sq. ft.) (° F.). The slope b, is the coefficient of linear regression of time (hours) on a functio of the temperature difference between the heat source (upper coppe plate) and the heat sink. Test specimen thickness, L, is expressed i inches. A more thorough description of the experimental and analyti cal procedure used may be found in the report of Bennett and co workers (*2*). The results, shown as averages by maturity groups, ar listed in table 4.

TABLE 4.—*Measured values of characteristic thermal conductivity, k for the rind and juice-vesicles components of Marsh grapefruit*

[Average of 10 runs for each maturity group]

Maturity group[1]	Rind thermal conductivity	Coefficient of variation	Juice-vesicle thermal conductivity	Coefficient of variation
	B.t.u. per (hr.) (sq. ft.) (° F. per ft.)	*Percent*	*B.t.u. per (hr.) (sq. ft.) (° F. per ft.)*	*Percent*
1	0. 1412	11. 9	0. 2562	9.
2	. 1371	13. 1	. 2810	6.
3	. 1398	6. 5	. 2848	5.
4	. 1426	9. 6	. 2822	9.
5	. 1379	7. 9	. 2539	4.

[1] Harvest dates for maturity groups are given on p. 2.

Because of the conditions discussed in the preceding section, tc simply call these values "thermal conductivity" would be a misnomer. Instead, it seems appropriate to use the term "characteristic thermal conductivity" to describe the heat transfer property obtained by the foregoing procedure.

Values of thermal conductivity, specific heat, and density thai characterize the total mass constituency are needed for computation of an apparent value of thermal diffusivity for the whole Marsh grapefruit by use of equation (8). Such values would not be accurate but they would provide a characteristic measure of the respective properties of the combined components. For this purpose, an apparent thermal conductivity for the whole fruit is calculated by utilizing the measured values obtained for the separate components. If the juice-vesicle and rind sections are assumed to be two hollow concentric spheres (fig. 6), the total resistance to heat transfer is

$$\Omega_t=\Omega_{12}+\Omega_{23} \qquad (10)$$

where, resistance through the juice vesicles is

$$\Omega_{12}=\frac{(r_2-r_1)}{4\pi k_{12}r_1r_2} \qquad (11)$$

through the rind is

$$\Omega_{23}=\frac{(r_3-r_2)}{4\pi k_{23}r_2r_3} \qquad (12)$$

and through both the juice vesicles and the rind is

$$\Omega_t=\frac{(r_3-r_1)}{4\pi k_{app}r_1r_3}. \qquad (13)$$

Equations (11), (12), and (13) can be combined and simplified to

$$k_{app}=\frac{k_{12}\,k_{23}r_2(r_3-r_1)}{k_{12}r_1(r_3-r_2)+k_{23}r_3(r_2-r_1)}. \qquad (14)$$

This technique is valid and is widely used for computing a single factor that denotes the heat transfer characteristic of a heterogeneous mass constituency consisting of several adjoining layers of different materials. Examination of values of apparent thermal conductivity, (table 5) indicates that the effect of the rind is almost negligible. A significant bias in favor of the juice-vesicle component is evident. When this factor is used in equation (8) to compute apparent thermal diffusivity, the inherent bias is reflected in the results.

The product of specific heat and density is a single factor that denotes the heat capacity per unit volume of a substance. Values given in table 5 represent the sum of partial capacities of the rind and juice-vesicle components based upon proportionate volumes of test fruit. The rind of Marsh grapefruit constitutes 30 to 40 percent

TABLE 5.—*Calculated values of apparent thermal conductivity and apparent thermal diffusivity for the whole Marsh grapefruit, based on measured thermal conductivity for the rind and juice-vesicle components*

Maturity group [1]	Heat capacity	Apparent thermal conductivity	Apparent thermal diffusivity
	B.t.u./cu.ft./° F.	*B.t.u./hr.ft./° F.*	*Sq.ft./hr.$\times 10^{-3}$*
1	46. 0	0. 2513	5. 5
2	45. 8	. 2755	6. 0
3	45. 8	. 2768	6. 0
4	47. 6	. 2764	5. 8
5	49. 2	. 2502	5. 1

[1] Harvest dates for the maturity groups are given on p. 2.

of its total volume. Its unit heat capacity is about half that of t juice vesicle and, therefore, is only 20 to 25 percent of the tot Consequently, the weighted heat capacity of Marsh grapefruit approximately 20 percent less than it would be if the effect of t rind were neglected. Specific heat and density (table 3) were us for the weighted computations.

Apparent thermal diffusivity values reported in table 5 are 37 percent greater than those reported in table 1. Part of this discrepan may be attributed to the reasons explained in the preceding secti (p. 14). However, most of the error seems to be caused by the bi introduced through use of apparent thermal conductivity in relati to weighted heat capacity. If specific heat and density for the who grapefruit are used as a basis for computing apparent thermal di fusivity, the results are comparable with those reported in table Hence, the error probably is a product of bias attributable to t technique used for obtaining the results.

Correlations

Statistical analyses were made to ascertain correlations, by maturit groups and over all maturity groups, between the following variable (1) Rind thickness, (2) rind thermal conductivity, (3) juice-vesic thermal conductivity, (4) effective thermal diffusivity, (5) rin moisture content, (6) juice-vesicle moisture content, and (7) specifi gravity. An analysis of all possible correlations over all maturit groups revealed five statistically significant correlations. In additio four other relations reflected a strong tendency to correlate, bu these were not significant at the 5-percent level of probability. (Se following tabulation.)

Variable correlation: [1]	*Correlatio coefficients (r)*[2]
Rind thickness on specific gravity	−0. 782
Rind moisture content on specific gravity	−. 591
Rind thickness on rind moisture content	. 486
Rind moisture content on juice-vesicle moisture content	. 405
Thermal conductivity of juice vesicle on specific gravity	−. 304
Thermal conductivity of rind on juice-vesicle moisture content	. 264
Thermal conductivity of rind on rind thickness	−. 255
Thermal conductivity of rind on rind moisture content	. 200
Rind thickness on fruit size	. 199

[1] All maturity groups combined.
[2] Critical value of r, 5-percent level, 0.273.

From these correlations a considerable interaction between rin thickness, moisture content of the rind and juice vesicle, therma conductivity of the rind and juice vesicle, and specific gravity i noted, with the specific gravity having the greatest influence. How ever, when maturity groups are considered separately, measure effective thermal diffusivity is found to correlate directly with specifi gravity and inversely with rind thickness (table 6). In the absence o a clarifying explanation, the seemingly wayward results of maturity group 5 must be attributed to experimental error.

TABLE 6.—*Correlation of measured effective thermal diffusivity on specific gravity and rind thickness for Marsh grapefruit by maturity groups*

Maturity group [1]	Correlation coefficients (r) [2]	
	Thermal diffusivity on specific gravity	Thermal diffusivity on rind thickness
1	0.5218	−0.3814
2	.6339	−.4690
3	.7914	−.5598
4	.8295	−.6084
5	−.1436	.4290

[1] Harvest dates for the maturity groups are given on p. 2.

[2] Critical value of r, 5-percent level, 0.602.

Another interesting result of this investigation of correlations is the effect of harvest date on certain of the physical properties. This effect was not evaluated statistically, but it is apparent from the data given in table 2. Results indicate that as the harvest season advances the fruit becomes more dense, its rind becomes thinner, and the moisture content of the rind decreases. Harvest date apparently did not affect moisture content of the juice vesicles. A notable tendency for measured effective thermal diffusivity to correlate with harvest date if maturity group 1 is omitted from consideration (table 1). This omission is considered valid because of experimental error encountered in maturity group 1, which caused the values to be significantly larger than those in the other four groups.

Temperature Distribution

Multiple regression analysis

As described under "Experimental Procedure," the temperature history of each test fruit was measured at ¼-inch intervals along the radius in the equatorial plane. The multiple regression technique of curve fitting was used to compute polynomial coefficients that express temperature distribution within the fruit as related to time for each maturity group. The model for the prediction equation is of the form

$$Y=a+b_1x_1+b_2x_2+b_3x_1^2+b_4x_2^2+b_5x_1^3+b_6x_2^3+b_7x_1x_2$$

where, in this example,

Y = the dimensionless temperature variable
x_1 = time
x_2 = position in the fruit
a = intercept
b_1 to b_7 = constant coefficients

when the temperature and position variables are expressed as normalized ratios, the resulting coefficients should be essentially equ
in all planes. Computed values for the respective maturity grou
are listed in table 7. The coefficient of variation among maturit
group means for temperature was 5.22 percent.

Temperature distribution during cooling does not differ statisticall
between maturity groups or between runs within a maturity grou
Consequently, for practical application, predicted values from any
the five groups will adequately describe temperature distribution withi
Marsh grapefruit during cooling with negligible surface heat transf
resistance. Figure 9 illustrates the internal temperature distributio
within Marsh grapefruit, initially at 85° F., being cooled in agitate
ice water at 35°. Data for similar curves may be calculated by sub
stituting the appropriate constant coefficients into the regressio
equation and solving for temperature ratio for any number of specifie
conditions.

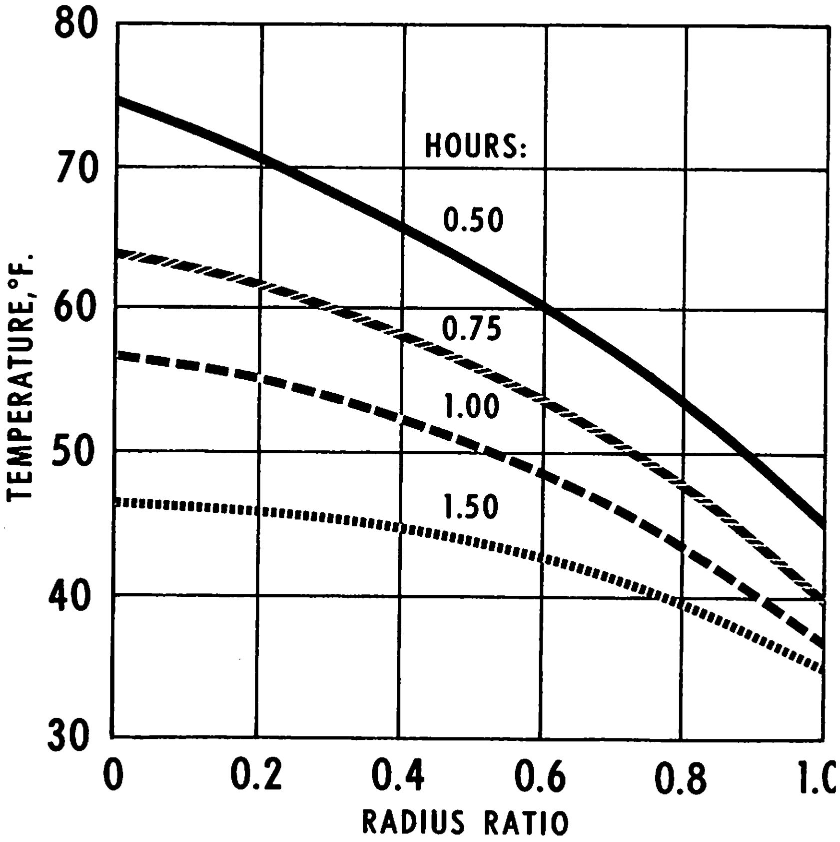

FIGURE 9.—Temperature response within a Marsh grapefruit, initially at 85° F cooled in agitated water at 35°. Calculated from prediction equation for maturity group 4.

TABLE 7.—*Intercepts and polynomial coefficients for temperature distribution in each of 5 maturity groups of Marsh grapefruit*

Maturity group [1]	a	b_1	b_2	b_3	b_4	b_5	b_6	b_7	Coefficient of variation
									Percent
1	1.2424	−1.1601	−0.4334	0.3973	−0.5532	−0.0562	0.1841	0.3852	13.96
2	1.2621	−1.1340	−.4159	.3647	−.1710	−.0460	−.1410	.3202	4.45
3	1.2326	−1.1063	−.4588	.3713	−.0786	−.0508	−.1801	.3093	3.80
4	1.2762	−1.2196	−.5117	.4540	−.0858	−.0731	−.1548	.3454	5.10
5	1.2436	−1.2058	−.3590	.4313	−.3293	−.0597	−.0247	.3243	4.60

[1] Harvest dates for the maturity groups are given on p. 2.

Cooling curves plotted from prediction data do not precisely co
form to actual temperature distribution patterns. The curve-fittir
analysis is an approximating procedure; hence, small irregularitie
whether actually present or experimentally induced, are not show
From figure 9 it appears that the temperature distribution from cent
to surface is smooth, with no sudden change in gradient between t
juice-vesicle and rind components. Actually, because of the relati
resistance of heat transfer, there is a noticeable change in gradient
the interface between the two components. This phenomenon is di
cussed in the following section.

Effect of rind on temperature distribution

Findings reported in earlier sections have shown that the thr
Marsh grapefruit rind properties, (1) thermal conductivity, (
specific heat, and (3) density, are not equal to the correspondi
juice-vesicle properties. Heat, during transient cooling, flows throu
these two different materials, the rind and the juice vesicles, alo
radial coordinates moving from center to surface (fig. 10).

According to laws of heat transfer, it is possible to have a disco
tinuity of temperature and of temperature gradient at the interfa
between the two materials. It has been shown that the therm
diffusivity of the two components is essentially equal. From Schneider
"Temperature Response Charts" (*8*), it is seen that the temperatu

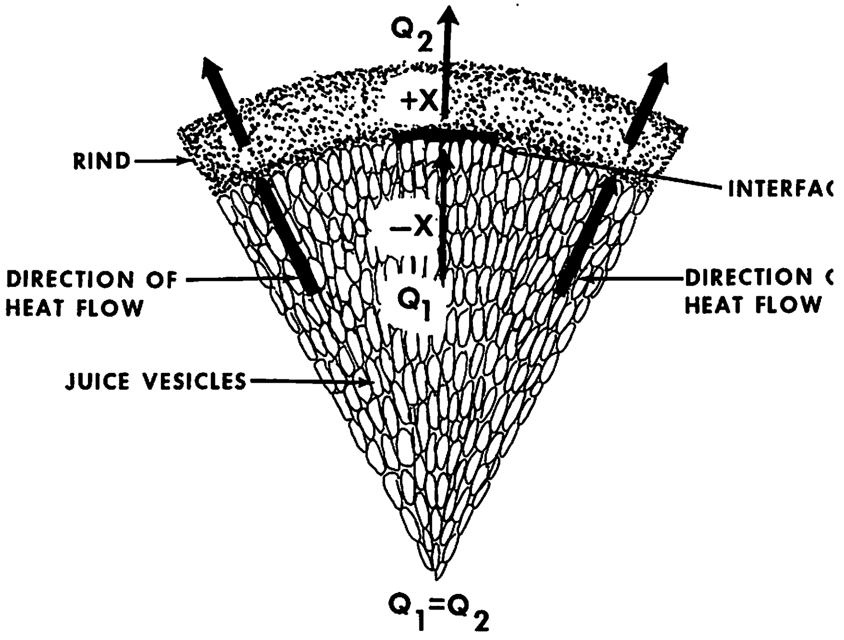

FIGURE 10.—Section of Marsh grapefruit illustrating the different properties
the rind and juice-vesicle components and the flow of heat through the interfa
between the two components.

at the interface of the two adjoining materials is continuous. Therefore, there is no change in rate of heat transfer through the boundary between the two materials. At an instantaneous time, the heat, dQ_1, that leaves the juice vesicles in the direction of positive x, is equal to the heat, dQ_2, that enters the rind. When these two elemental heat quantities are equated, the equation is

$$k_1\left(\frac{dt}{dx}\right)_1=k_2\left(\frac{dt}{dx}\right)_2. \qquad (15)$$

From table 4 it is noted that $k_1 \neq k_2$. Therefore, to satisfy the condition of equation (15), the temperature gradient $(dt/dx)_1$ does not equal the temperature gradient $(dt/dx)_2$. As Gröber and coworkers (*5*) observed: "Since there is a discontinuity in k at the point considered, it follows therefore that there must also be a discontinuity in the temperature gradient, and this is true not only for the steady state but also for the unsteady state."

The cooling of Marsh grapefruit involves unsteady state heat transfer through the juice-vesicle and rind components. Since the properties of these components are different, there is a difference in the temperature gradient between the two. There is, therefore, a "break" in the temperature distribution curve at the interface between the two materials. This "break" is not noticeable when the temperature distribution is plotted from the prediction equation. However, when raw experimental data is plotted, the response is evident. This response, representing an average of 10 test runs from maturity group 4, is shown in figure 11.

Linear regression analysis

Solution of the regression equation containing the polynomial coefficients listed in table 7 produces a nonlinear response when plotted on rectangular coordinates. For specified values of time and distance ratio, a family of curves is generated. When these data points are plotted on semilogarithmic coordinates, the result will be a family of straight lines whose slopes are a function of time, position, product geometry, and heat transfer property. This linear response may also be evaluated by converting the dimensionless temperature variable to logarithms and computing a linear regression. If there is a close "fit" of the data points to a straight line in the graphical analysis, the results of the two methods will be comparable; i.e., essentially equal slopes.

A linear regression of temperature as related to time—at the center, at one-half the radius, and at three-fourths the radius—was computed for each Marsh grapefruit test run. The general form of the equation is

$$Y=a+\beta x_1$$

where, for this application, Y is $\log_e T$, a is the intercept, and β is the constant coefficient corresponding to the slope. The slope β is used in equation (5) for computing effective thermal diffusivity. The intercepts and coefficients are listed in table 8 as maturity-group means. The linear response of maturity group 4 is shown in figure 12.

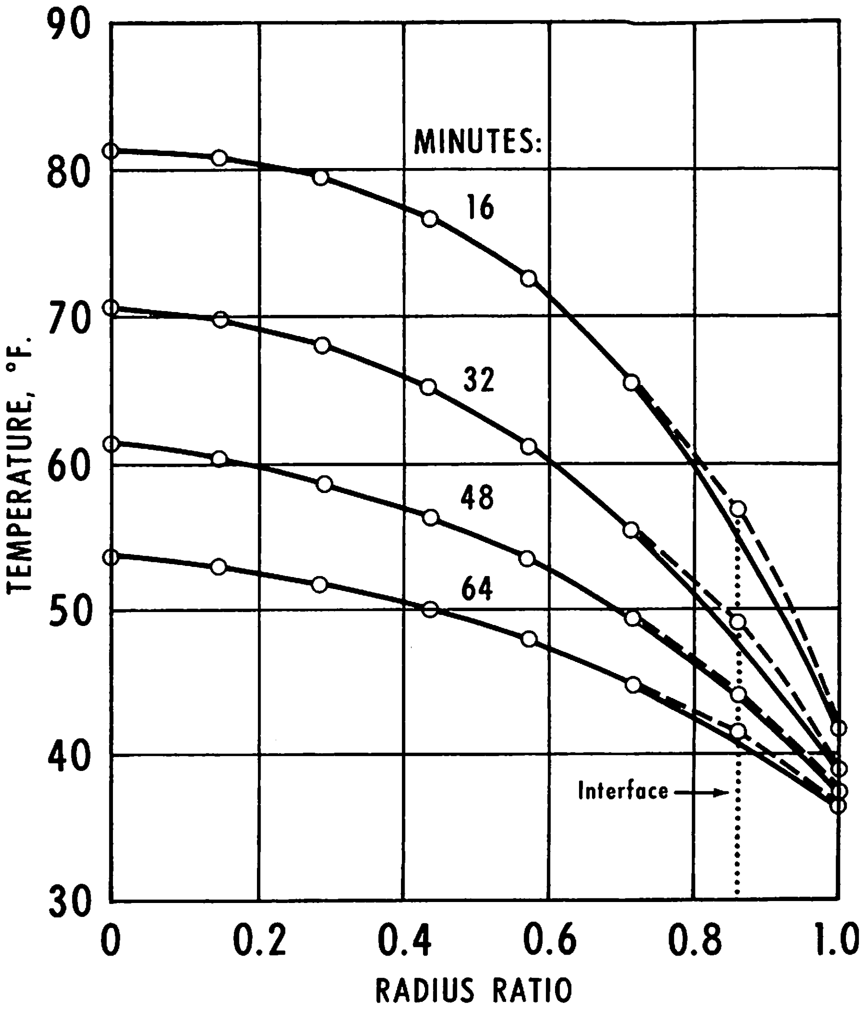

FIGURE 11.—Internal temperature distribution in Marsh grapefruit plotted from raw experimental data recorded during transient cooling test runs. Dotted lines show discontinuity in temperature gradient at interface. Average of 10 test runs from maturity group 4.

Pflug and coworkers (*7*) plotted the actual product temperatures on a logarithmic scale so that the change of temperature per unit change of time could be read directly. This procedure provides a simple and easily understood way of showing the temperature response within a product during cooling. It, however, has the disadvantage of applying to specific conditions of the product and the surrounding fluid temperature. In practice, these conditions are

TABLE 8.—*Linear temperature response parameters for Marsh grapefruit cooled in agitated ice water*

[Maturity group means]

Maturity group and location of response[1]	Slope	Intercept	Coefficient of variation	Correlation coefficient
1:				
A	−1.23	0.29	5.44	−0.96
B	−1.35	.11	5.97	−.95
C	−1.47	−.32	6.67	−.93
2:				
A	−1.17	.29	5.12	−.97
B	−1.25	.20	5.42	−.98
C	−1.33	−.12	5.81	−.98
3:				
A	−1.14	.26	5.00	−.97
B	−1.22	.17	5.39	−.97
C	−1.27	−.16	5.65	−.96
4:				
A	−1.17	.29	5.10	−.99
B	−1.26	.19	5.45	−.99
C	−1.34	−.14	5.94	−.97
5:				
A	−1.23	.28	5.47	−.96
B	−1.34	.19	5.96	−.97
C	−1.46	−.15	6.55	−.95

[1] Harvest dates for maturity groups are given on p. 2. A, Center of fruit; B, one-half of radius; C, three-fourths of radius.

generally confined to a narrow range, and two curves of upper and lower limits may be used to encompass a band of normally expected conditions. Temperature-time curves for specified points within the fruit may be plotted from the empirically based coefficients listed in table 7. The parameters listed in table 8 may also be used to plot temperature response at the locations shown.

Mass-average temperature

The concept of mass-average temperature is receiving increased acceptance among engineers and scientists engaged in the design of, or research toward, development of refrigeration systems for cooling (or heating) perishable food products. This concept is particularly important with rapid precooling where there is likely to be a temperature gradient from the center to the surface of the substance being cooled.

The amount of heat stored in or released from a substance in a given time is ascertained by measuring the temperature. If the temperature throughout the substance is uniform, measurement may be made without regard to location within the substance. If the temperature is not uniform, however, an average temperature must be used. If the gradient is linear, an average is easily obtained. When the gradient is not linear, which is usual, a mass-average temperature, based on the nonlinear temperature distribution as related to time, should be evaluated.

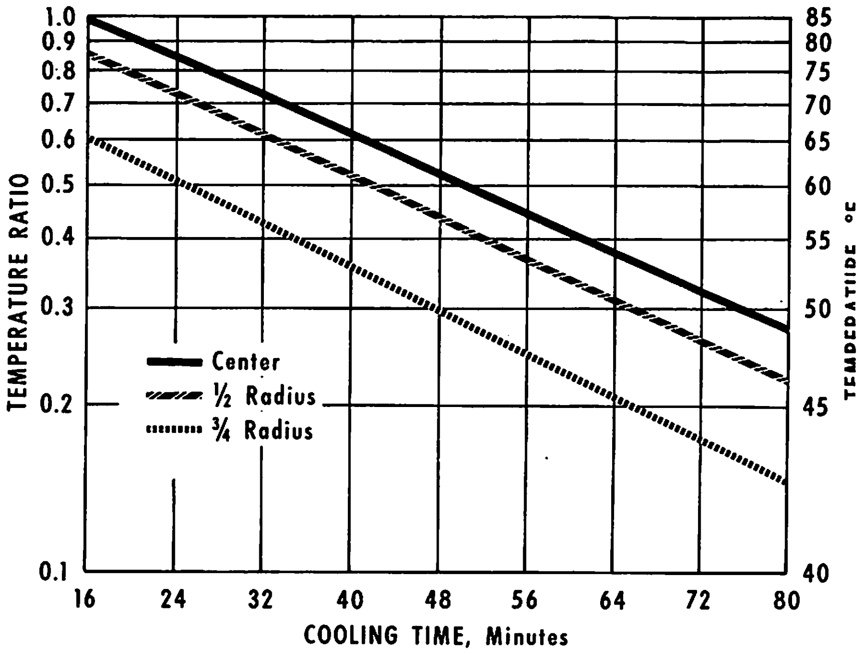

FIGURE 12.—Linear temperature response of Marsh grapefruit, initially at 85° F, cooled in agitated ice water at 35°. Fruit harvested in February 1965 (maturit group 4).

When Marsh grapefruit at room temperature is immersed in a bath of agitated ice water, its surface temperature suddenly becomes essentially equal to the temperature of water. Initially, there is a steep temperature gradient near the surface of the fruit. As cooling progresses, the gradient moves toward the center and diminishes as it moves inward. Eventually, the gradient vanishes. The fruit is then at the temperature of the water throughout. This characteristic relation of surface to internal heat transfer is discussed in the section on evaluation of effective thermal diffusivity. The application of this phenomenon in the evaluation of the magnitude and location of mass-average temperature during rapid precooling of Marsh grapefruit is described in this section.

The method of Smith and Bennett (*9*) for evaluating the mass-average temperature within a substance during transient cooling makes use of the expression for internal temperature distribution as related to time. By substituting specified time values into the polynomial prediction equation, they obtained a set of equations (one for each time) that express the temperature ratio, $Y(R)$, in terms of the radius ratio, R. The equation takes the form

$$Y(R)=a+bR+cR^2+dR^3 \qquad (16)$$

where a, b, c, and d are the constant coefficients of R.

The mass-average temperature of a substance denotes a measure of the heat contained in the substance, above some reference level, at any time during heating or cooling. The heat content of a sphere, when $R=1$, was expressed by Smith and Bennett (*9*) in the form

$$Q=\frac{K\,Y_{ma}}{3} \tag{17}$$

where K is a constant for $4\pi\ \rho\ c_p$. By equating this equation to an equation that expresses the same heat content in terms of the temperature distribution along the radial coordinate, they obtained a function that expresses temperature ratio at the mass-average point for a specified time.

The heat content of the substance is a direct function of its volume, or mass. The volume of an oblate spheroid is less than the volume of a sphere whose diameter is equal to the major axis of the spheroid. Based on the average equatorial and polar diameters of the 50 test fruit used in this study, the volume coefficient of Marsh grapefruit is 0.883. Volume coefficient is the ratio of the volume of the average fruit specimen used in this study to the volume of a sphere whose diameter is equal to the equatorial diameter of the fruit, which means that the heat content of Marsh grapefruit is 88.3 percent that of a corresponding sphere. Introduction of this volume coefficient into equation (17) yields a measure of the heat content of an oblate spheroid whose dimensions correspond to the representative fruit of this study; that is,

$$Q=\frac{0.883\ K\,Y_{ma}}{3}=0.2943\ K\,Y_{ma}. \tag{18}$$

When equation (18) is equated to the expression of heat content in terms of the temperature distribution along the equatorial plane of Marsh grapefruit, the equation obtained is

$$Y_{ma}=1.132a+0.848b+0.679c+0.566d. \tag{19}$$

Substitution of the above constant coefficients into equation (19) yields the solution of mass-average temperature ratio for the specified time. The point on the radius where this particular mass-average temperature occurs may be found by either graphical or analytical solution of equation (16).

Application of this method for maturity group 4 takes the following procedure:

Insert the appropriate polynomial coefficients from table 7 into the model equation and solve for temperature ratio, Y_{ma}, in terms of radius ratio, R. For a cooling time of 30 minutes (0.5 hours), compute

$$Y(R)=0.7708-0.3390R-0.0858R^2-0.1548R^3 \tag{20}$$

Thus

$$a=0.7708;\ b=-0.3390;\ c=-0.858;\ d=-0.1548.$$

Substitute the respective coefficients into equation (19) an solve for Y_{ma}.

Result:

$$Y_{ma}=0.4391.$$

Problem: Initial fruit temperature is 90° F.
Surface temperature during cooling is 35°.

Solution:

$$Y_{ma}=\frac{t-ts}{t_i-ts}$$

$$0.4391=\frac{t-35^\circ}{90^\circ-35^\circ}$$

$$t=59.15^\circ\mathrm{F}.$$

The point along the radius in the equatorial plane where th mass-average temperature occurs can be found by substitutin the value obtained for Y_{ma} into $Y(R)$ of equation (20) an solving for R. The result is

$$R=0.699.$$

This process is one of trial and error. Values of predicted magnitud and location of mass-average temperature may be similarly compute for other cooling times. Values so computed for maturity group 4 ar illustrated in figure 13.

CONCLUSIONS

The effective thermal diffusivity of Marsh grapefruit may be a curately evaluated by using the first-term approximation of Fourier Sine-Series temperature response and applying Smith's correctio factor.

There is no appreciable difference in the thermal diffusivity of th rind and juice vesicles when calculated from separately measure values of thermal conductivity.

The thermal conductivity of the rind of a Marsh grapefruit is abou one-half that of the juice vesicles. When the thermal conductivity measured by the method cited in this bulletin, a small convectio component of heat transfer causes the resulting values to be highe than if conduction heat transfer alone were involved.

Apparent thermal conductivity, computed by summing the r sistances of the rind and the juice-vesicle components, does not provid a meaningful measure of the heat transfer characteristic of a whol Marsh grapefruit unless the error introduced by inherent bias i corrected.

Findings have shown a significant interaction between rind thick ness, moisture content of the rind and juice vesicles, thermal cor ductivity of both rind and juice vesicle, and specific gravity; and also that as the harvest season advances from October through Ma fruit becomes more dense, its rind becomes thinner, the moistur content of the rind decreases. These findings are consistent with th

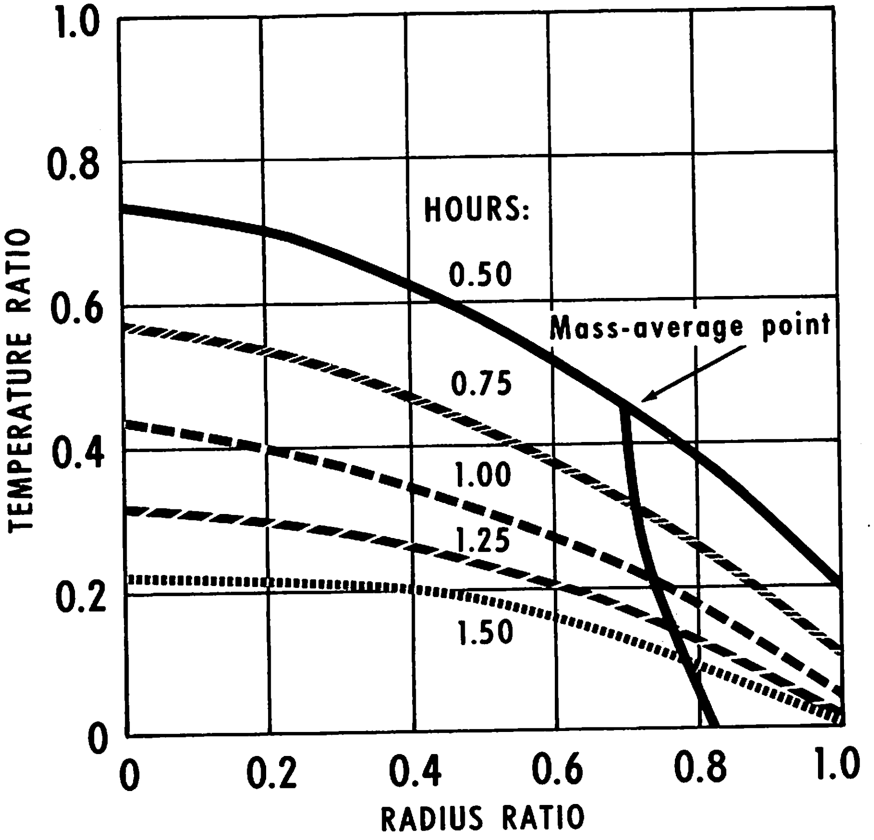

FIGURE 13.—Magnitude and location of mass-average temperature of Marsh grapefruit cooled in agitated ice water. From temperature response of maturity group 4.

observed increase in effective thermal diffusivity in relation to length of harvest season.

The effect of time of harvest on experimental temperature distribution was not considered statistically significant. However, the analytical technique was probably not sufficiently critical to detect the effect of harvesttime on temperature distribution.

The influence of the greater heat transfer resistance through the rind than through the juice vesicles is noticeable in experimental data recorded during transient cooling. This factor could possibly have a slight effect on cooling rate, but it would not be sufficient to be considered of significance for commercial application.

Because Marsh grapefruit is an oblate spheroid, its mass-average temperature is slightly nearer to the center than it would be if it were a perfect sphere.

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